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## LINEAR MODELING OF A SELF-OSCILLATING PWM CONTROL LOOP

There are well established methods of creating linearized versions of PWM control loops to analyze stability and to create fast “behavioral” models of the systems that do not rely on evaluating behavior during each individual cycle. Unfortunately, even more advanced modeling techniques such as state-space averaging are difficult if not impossible to apply to self-oscillating loops such as simple hysteretic control.

It is nonetheless possible to construct fairly accurate models of self-oscillating converters using the technique to be disclosed. This technique is suggested by the analysis of a switching audio amplifier in an AES paper by Bruno Putzeys<sup>1</sup>.

### Goal

It is the primary intent of this work to generate a procedure for creating a linearized model that will allow:

- a) frequency domain simulation of the control loop at baseband frequencies.
- b) fast time domain analysis of transient response (e.g. load regulation, step response)
- c) optimization of networks and evaluation of first order sensitivities to component variation.

It is a given that although the system should model the closed feedback of the self-oscillating loop, the goal is to analyze not the oscillation, but the response of the system in all respects *other than the oscillation*. This paper will demonstrate that the approach can accurately show the oscillatory nature of the system, but that the oscillation can be “turned off” and the system evaluated without regard to the high frequency carrier.

The close matching between a full cycle-by-cycle simulation and the linearized model will be demonstrated.

### Simple System Model

The systems to be modeled are often used either for switching regulators or in switching audio amplifiers, and typically amounts to placing a comparator within a feedback loop. The comparator, often with hysteresis, drives a power switching node rail-to-rail, and an output filter reconstructs a DC or audio frequency signal from the switching output. This output, typically including some small ripple, will be fed back to the comparator in appropriate phase for a conventional negative feedback loop. But the nature of the hysteretic input or just arbitrarily high gain of the overall feedback control loop causes the system to oscillate continually.

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<sup>1</sup> Putzeys, Bruno, “Simple Self-Oscillating Class D Amplifier with Full Output Filter Control” Presented at the 118 AES Convention, May 2005, Preprint [ [http://www.ciaudio.com/ucd\\_aes.pdf](http://www.ciaudio.com/ucd_aes.pdf) ]

Figure 1 shows a simple switching regulator composed of three basic blocks. At left is a controller featuring an ideal (zero delay, near infinite gain) comparator, a digital delay, and an output stage. This is followed by an output filter LC network, shown here without its associated parasitic elements other than the series resistance of the inductor. A feedback network connects both to the switching output of the controller block and to the output of the LC network. In the following example, this will be analyzed as a 20MHz buck converter.

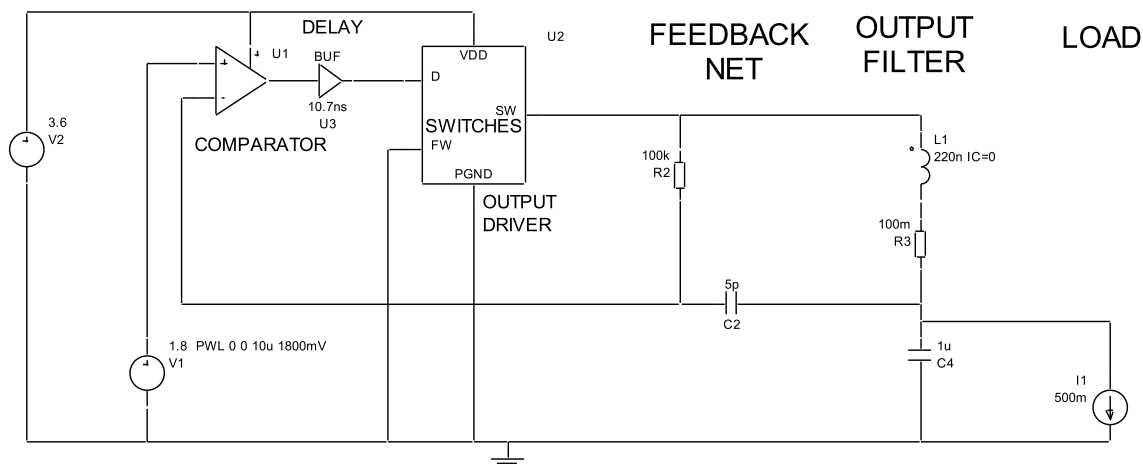


fig.1 Simple Self-oscillating Buck Converter

Figure 1 is a very simple model of the type of systems that are of interest. It was chosen not because it is a good way to build a self-oscillating loop, but rather because it is simple and compact. It is the goal to be able to analyze systems that are more sophisticated and include parasitic elements. The limitations of this system allow for easier verification of results. Note for instance that that the system shown has no DC feedback from the final output, only the switching node. The loop therefore cannot correct for the output impedance error of  $100\text{m}\Omega$  R3.

Although figure 1 shows a comparator with no hysteresis, it can be shown that hysteresis and delay are in many ways equivalent in self-oscillating loops. An increase in hysteresis can be correlated to an equivalent increase in delay. In this linearized model, the hysteresis will be ignored and changes in linear delay will be a surrogate for hysteresis.

This is not yet a very good buck regulator, as we can verify with a transient simulation. We increase the reference voltage V1 from 0 to 1.8V in the first  $10\mu\text{s}$ , and after a while we hit the output with a 500mA load pulse. We can see that there is a finite load regulation (the  $100\text{m}\Omega$  series resistance of the inductor), and some ringing. Trying two values for feedback resistor R2 gives different amounts of overshoot. But changing the value of R2 by a factor of 2 didn't change the switching frequency appreciably (from 19.8MHz to 20.2MHz, about 2% change). It seems to have mainly affected the closed loop damping, but not the small signal bandwidth.

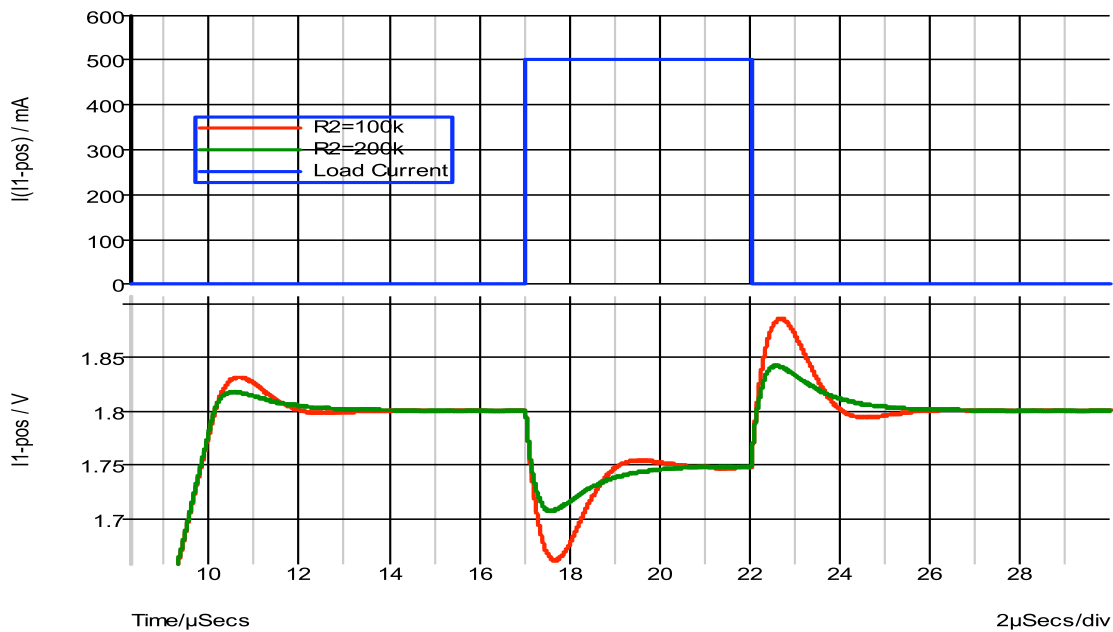


fig. 2 Response of buck regulator; settling and load step

A simplified model of this system, for simplicity, would replace the controller at left with a linear system that could be analyzed without switching. The comparator, delay, and switch can in fact be replaced by a linear amplifier with a finite output impedance and a characteristic delay. The difficult question: how much gain does this system seem to have?

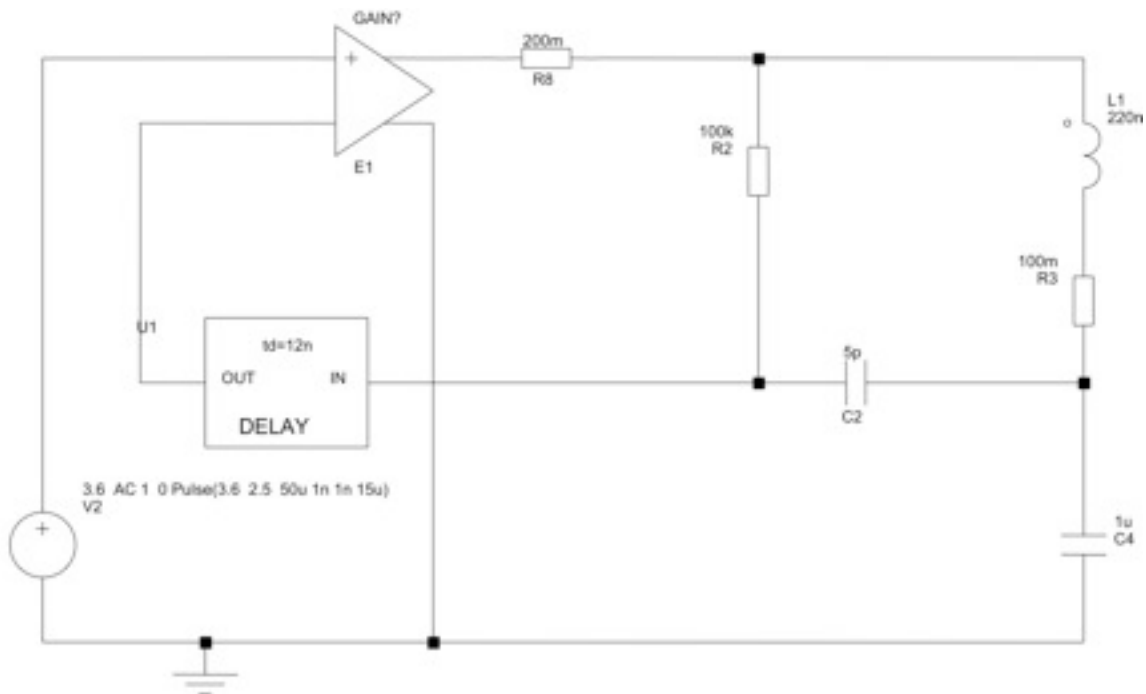


fig 3. Linearized System

## Determining Gain of the Loop

In a classic PWM modulator with a ramp compared to a control voltage, the effective gain is inversely proportional to the swing of the ramp. (Going from zero duty cycle to full duty cycle requires the control voltage to swing from just below the ramp to just above it.) One way to look at self-oscillating schemes is that they essentially have a ramp (ripple) at the input, created in this case by the switching waveform and  $R2/C2$ . The smaller this ramp, the higher the effective gain. Since the waveform at the comparator input is not strictly a ramp or 'triangle wave', the small signal gain is more accurately described as inversely proportional to the ripple  $dV/dt$  at the time the comparator is tripped.

Since the feedback network is generally a lowpass, the magnitude of the effective ramp is related to the attenuation from the switching node to the feedback node. By assuming the switching node is a square wave, and make some often reasonable assumptions on the resulting  $dV/dt$ , Putzys' generalized that the effective gain is essentially magnitude of the feedback attenuation at the switching frequency with a minor fudge factor. But the assumptions presume that the comparator input is dominated by the fundamental of the square wave, and that the slope at which this sinusoid crosses zero is the applicable slope. Examination of real waveforms in practical systems lends little credence to these assumptions.

Putzys' small signal modeling often shows a high frequency "bump" near the supposed oscillating frequency. It seemed a little odd that the linear system didn't oscillate, until it was realized that he did not simulate any delay or hysteresis associated with his modulator/power stages. This suggested an alternate strategy: if the linear model can be made to oscillate like the target system, the model should accurately be in a small signal condition that is similar to the self-oscillating system.

Real oscillators must meet two criteria. First, the gain around the loop is 360 degrees at the frequency of oscillation, and the loop gain is precisely unity. In practice, oscillators have a gain set higher than unity, such that the magnitude of the resonance will increase (exponentially) with time. But real oscillators include some nonlinearity, such that as the magnitude of the oscillation increases, the gain effectively drop. The fixed output magnitude of a switching system or a clipped amplifier will often provide the reduction in gain needed to bring the system back to unity loop gain.

So it is reasonable that the system can be modeled by determining the gain needed to create the conditions for a stable oscillation around the bias point. This means that the gain provided by our amplifier should be precisely enough to compensate for the attenuation of our output filter, a criteria correlated to, and actually simpler than that of Putzys.

This simplification gives excellent correlation in all systems yet modeled. If we consider the case of the circuit of figure 1, we can measure this attenuation with a straightforward AC analysis. Sweeping an AC source at the switch node and plotting attenuation through the filter (including both the output filter and the feedback network, we get attenuation of 42dB and 36dB for the cases of 200k and 100k feedback resistors for  $R2$ . This corresponds to loop gains of about 120 and 60 (42 and 36dB).

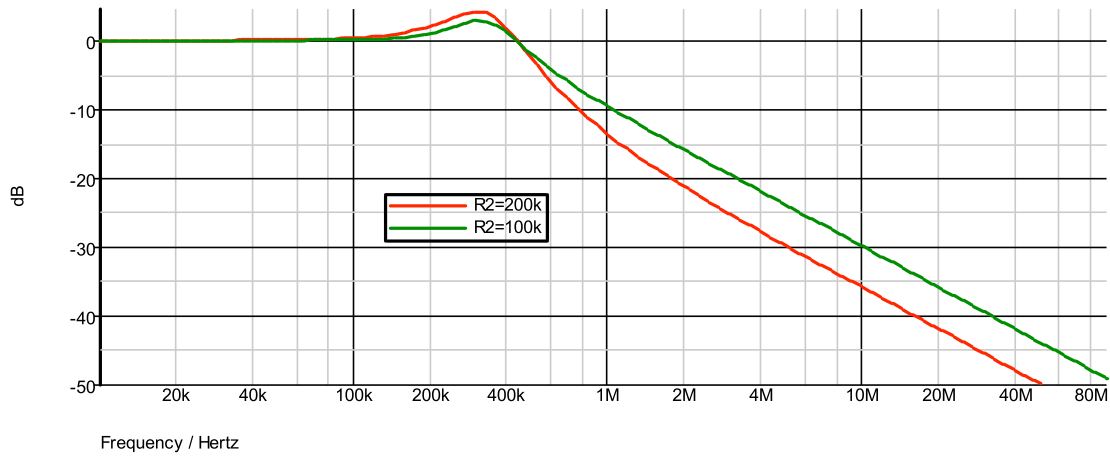


fig. 4 Attenuation of feedback network

We can plug these numbers back into the linear circuit of figure 3 and look at the closed loop response. Figure 5 shows both a nominal rolloff and sharp peak around 20MHz. The delay element was adjusted to pull this peak frequency to match the 20MHz switching frequency. If this response were truly oscillatory, one might the peak should go to infinity. In practice, the loop gain and phase must be set precisely for simulation to show this. In fact, the highest peaking I obtained in simulation was about 180 dB, but to see that, one had to sweep the frequency quite finely, as the magnitude dropped by 50 dB if one was only 1Hz off resonance. And the precision of the gain adjust was such that changing the gain by 0.05 ppm was enough to degrade the peak by 40dB. *But we aren't really interested in a precise linear model of this oscillation!.* This is a means to adjusting the gain and phase suitably realistically for modeling the response away from the oscillation point. We just want at this point to see that there is a sharp peak that indicates that we have properly identified the gain and delay elements to be very close in practice to the point of oscillation.

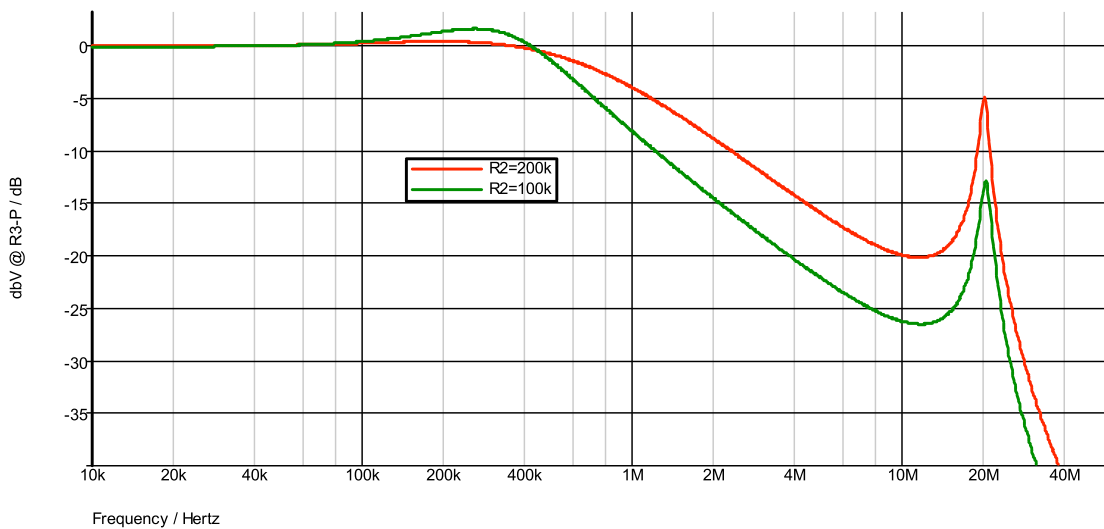


fig. 5 Closed loop response of linearized system

We do not want our linear model to oscillate, but we want the response to otherwise mimic reality. If we remove the delay element, our linear model loses (in this case) about 90 degrees of phase lag at 20MHz, and the peaking/oscillation goes away. But the delay has no appreciable impact at lower frequencies; the 90 degree phase lead we just added is less than 2 degrees of phase error at the -3dB point of 500kHz. Figure 6 shows the change in closed-loop response in the case of the poorly damped system with  $R2=100k$ . And unlike how critical it may be to get the gain approximated just right to get the system to look oscillatory, the open loop gain was also adjusted  $\pm 6$ dB with little appreciable effect on the low frequency loop response.

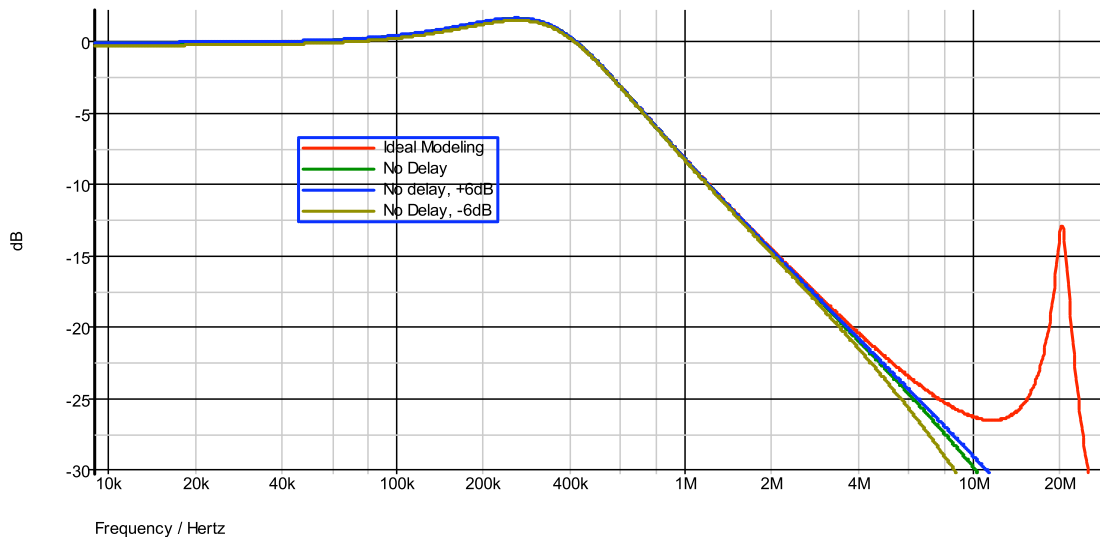


fig. 6 Linearized system; delay removed and loop gain varied

## Verifying the linear system

In order to verify the linearized model, we need only replace our comparator/delay/switch model with the linearized gain block (gain and output impedance, but ignoring delay), and we can re-simulate the turn-on settling and load step of figure 2. Figure 7 shows the results, with the upper graph being the full simulation reported in figure 2, and the lower graph the same conditions simulated with the linearized model.

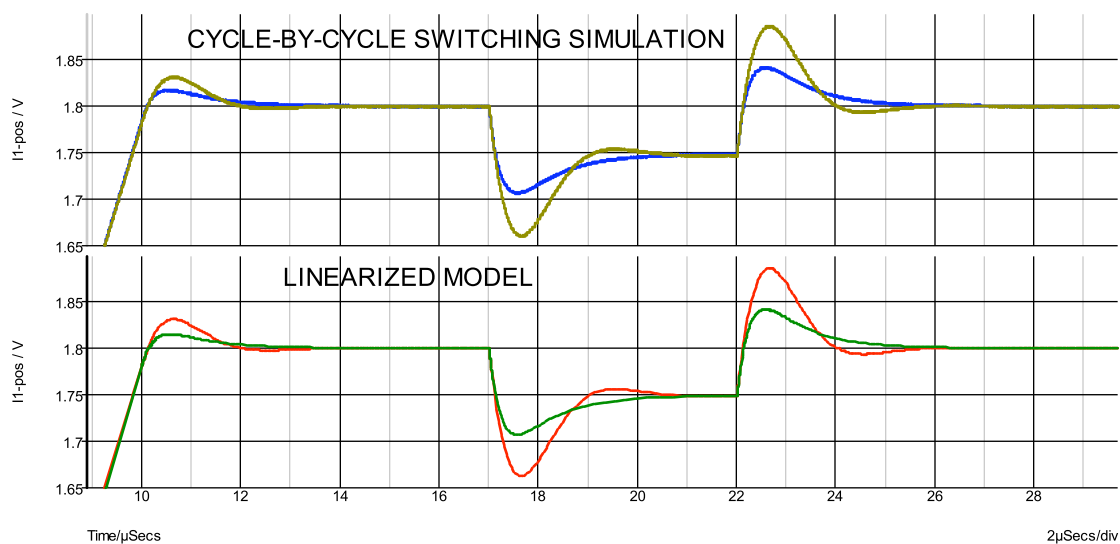


fig 7. Linearized simulation compared to full simulations of fig. 2, for the cases of  $R2=100k$  and  $R2=200k$

## Procedure

1. Simulate or measure target system; measure frequency of oscillation, capture a transient behavior for later verification.
2. Measure the attenuation of the filter from switch node to feedback node at the switching frequency.
3. Set the magnitude of the linear amplifier to equal the attenuation value, and set its output impedance to be that of the output switches.
4. Adjusting the delay element to get the proper frequency of operation, verify that the system with linear amp and delay would self-oscillate at the frequency measured in 1 above. (Some adjustment of gain may be needed)
5. Removing the delay, test the full loop under the conditions of (1) above and verify similar behavior.

Now that the model exists, standard techniques to play with loop compensation etc. can be used to optimize the system. Bode analysis can be used (breaking the linear loop to isolate open loop and closed loop responses). Output impedance of the system can be examined in either a transient analysis or an AC analysis.

If substantive changes have been made, it may be desirable to take the modified system back to step (1) and recreate a new linear model, but as shown above, even moderate changes in loop gain typically are not critical to the closed loop response.